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Design of Multi-Region Pressure Vessels Using Maximum Shear Theory

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LINCOLN LABORATORY

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ABSTRACT

A method is outlined for multi-region pressure vessels design calculations using the maximum shear theory. This treatment is employed due to the simplicity of the method and because the results are quite conservative for both ductile and brittle materials.

A procedure for obtaining an optimum design has been given for a desired percentage of auto-frettage on the inner wall of the pressure vessel. A computer program has been written in Fortran II language and the various design possibilities have been executed by IBM-1620 computer.

Accepted for the Air Force
Joseph R. Waterman, Lt. Col., USAF
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NOMENCLATURE

E = Young's modulus

K = $(r - r_o)/(r_1 - r_o)$ ratio of the assumed inelastic region over the wall thickness of the ring

n_1 = Outside to inside radius ratio of first ring

n_2 = Outside to inside radius ratio of second ring

n_k = Total outside to inside radius ratio

N_k = Maximum total outside to inside radius ratio

p_o = Internal pressure psi

p_1 = External pressure psi

r_o = Inner radius

r_1 = Outer radius of first ring

r_2 = Outer radius of second ring

u = Radial deformation

u_o = Radial deformation of inner fiber

u_{o1} = Radial deformation of outer fiber of first ring

u_1 = Radial deformation of inner fiber of second ring

δ_o = Radial interference first ring

δ_1 = Radial interference between first and second ring

λ = $(r/r_o)^2$ ratio of the radius in the elastic range over the inside radius to the second power

λ_x = x subscript denotes the percentage of the auto-frettage

σ_t = Tangential stress

σ_r = Radial stress

σ_y = Yield stress

τ = Shear stress

μ = Poisson's ratio

I. Introduction

The purpose of this paper is to outline a method of designing a two-region pressure vessel using the maximum shear theory. This method was chosen because of its simplicity. Moreover, the results obtained in this way are quite conservative for both ductile and brittle materials. An attempt is also made to allow a certain percentage of auto-frettage[1] on the inner wall of the pressure vessel.

II. General Stress Relationship

The stresses in a thick-walled cylinder can be represented by the Lame-Clapeyron[2] equation.

$$\sigma_t = \frac{r_o^2 p_o - r_1^2 p_1}{r_1^2 - r_o^2} + \frac{r_o^2 r_1^2 (p_o - p_1)}{r^2 (r_1^2 - r_o^2)} \tag{1}$$

and

$$\sigma_r = \frac{r_o^2 p_o - r_1^2 p_1}{r_1^2 - r_o^2} - \frac{r_o^2 r_1^2 (p_o - p_1)}{r^2 (r_1^2 - r_o^2)} \tag{2}$$

where σ_t and σ_r represent the tangential and radial stress; r_o and r_1 denote the inner and outer radii of the cylinder, respectively; p_o and p_1 denote the inner and outer pressures, respectively.

Utilizing the Tresca criterion of yielding, i.e.,

$$\sigma_t - \sigma_r = \sigma_y \tag{3}$$

and substituting the Eqs. (1) and (2) into (3) for $r = r_o$, where σ_y is the largest, elastic breakdown begins on the internal surface at the pressure of

$$p_o = \frac{\sigma_y (n_1^2 - 1) + 2n_1^2 p_1}{2n_1^2} \tag{4}$$

where $n_1 = r_1/r_o$.

This works very satisfactorily within the elastic breakdown limit, but often one will find that p_o is too small to be of value in high pressure vessel design. To increase the pressure limits, auto-frettaging is utilized. Auto-frettage is originally defined as stretching the inner fiber of thick-walled cylinders beyond the elastic breakdown limit. The resultant effect upon release of the overstrain pressure will be that the outside layers will squeeze the inner layer, thus inducing a tangential residual stress that is compressive at the bore. Where multiple ring systems are concerned, the "self-hooping" effect (literal French translation for the term auto-frettage) is developed by the interferences between the consecutive rings. Auto-frettage should now be defined as stress in the inelastic region.

III. Shear Theory

The maximum shear theory which is the most conservative, states:

$$\tau = \frac{1}{2} \sigma_y \tag{5}$$

substituting Eqs. (1), (2), and (5) in Eq. (3) gives

$$\tau = \frac{r_1^2}{r^2} \frac{(p_o - p_1)}{(n_1^2 - 1)} \tag{6a}$$

Thus, Eq. (6a) satisfies all possible conditions where τ is maximum for the smallest r in the elastic range.

IV. Design Procedure for Two-Region System

Another expression for Eq. (6a) for a two-region system is

$$\tau_1 = \frac{n_1^2(p_o - p_1)}{r^2/r_o^2(n_1^2 - 1)} \qquad \text{first ring} \qquad (6b)$$

and

$$\tau_2 = \frac{n_2^2 p_1}{n_2^2 - 1} \qquad \text{second ring} \qquad (6c)$$

where $n_1 = r_1/r_o$ and $n_2 = r_2/r_1$, τ_1 and τ_2 are the shear stresses of the inner and outer cylinder. In Fig. 1, r represents the radius of the circle in the elastic range. The amount of auto-frettage can be expressed as a non-dimensional unit

$$\frac{r - r_o}{r_1 - r_o} = K$$

or

$$\frac{r}{r_o} = K(n_1 - 1) + 1$$

or

$$\lambda = K^2(n_1 - 1)^2 + 2K(n_1 - 1) + 1 \qquad (7)$$

where

$$\lambda = \left(\frac{r}{r_o}\right)^2$$

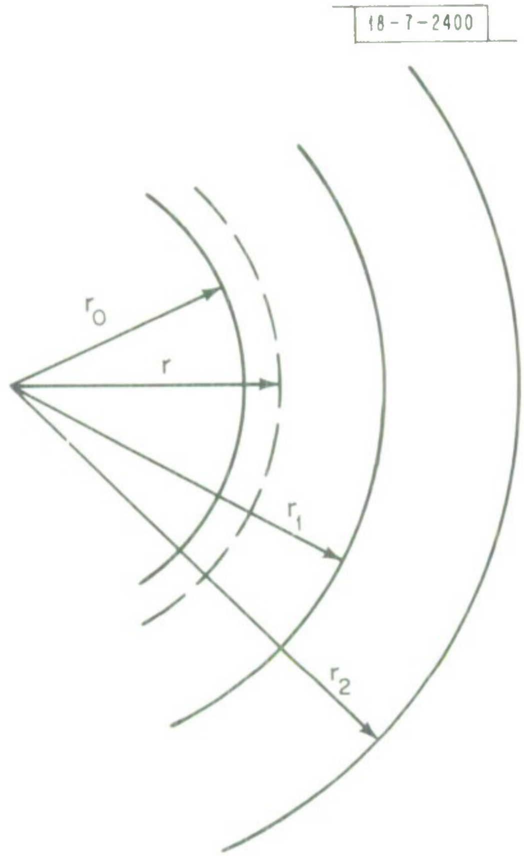


Fig. 1

Substitution of Eqs.(6c) and (7) into (6b) and expressed for n_k, will give an expression of the form

$$n_k^2 = \frac{n_1^4}{\lambda_x(n_1^2 - 1) + (1 - p_o/\tau) n_1^2} \tag{8}$$

where

$$n_k = \frac{r_2}{r_o}$$

V. Reverse Yield

This form of yielding occurs if the outside pressure p_1, on the inner cylinder, develops a hoop stress on the inner fiber exceeding the yield strength of the material. This condition can exist during assembly or during pressure release where $p_o = 0$ and thus

$$\sigma_t = \sigma_y$$

or

$$\sigma_{y1} = \frac{-2n_1^2 p_1}{n_1^2 - 1} \tag{9}$$

For the second ring, Eq. (3) can be expressed as

$$\sigma_{y2} = \frac{2N_k^2 p_1}{N_k^2 - n_1^2}$$

$$-\sigma_{y1} = \sigma_{y2}$$

or simply after solving for N_k

$$N_k = n_1^2$$

or simply

$$n_2 = n_1$$

VI. Interferences

Having determined all dimensions for a two-region pressure vessel by the described method, the following conditions are imposed to add to the safety of the design.

a. The inner surface of the first ring (whether at pressure under maximum conditions or employed as support for a die) does not undergo a dimensional change. This is true where the inner part is small or where this is made out of very brittle materials.

b. Each following ring does not undergo a dimensional change after assembly.

The general equation for the elastic radial deformation, u, at a radius, r, in a thick-walled cylinder is given as

$$u = \frac{1 - \mu}{E} \; \frac{p_o - n^2 p_1}{(n^2 - 1)} \, r + \frac{(1 + \mu)}{E} \; \frac{r_1^2 (p_o - p_1)}{r(n^2 - 1)} \qquad (10)$$

μ and E denote Poisson's ratio and Young's modulus, respectively. n is the ratio between the outer and inner radii. The radial deformation for the smallest radius of the innermost cylinder is then

$$u_o = \frac{1-\mu}{E} \frac{p_o - n_1^2 p_1}{n_1^2 - 1} r_o + \frac{1+\mu}{E} \frac{r_1^2 (p_o - p_1)}{r_o (n_1^2 - 1)}$$

or simply

$$\delta_o = r_o \frac{[p_o(1 - \mu + n_1^2 + \mu n_1^2) - 2n_1^2 p_1]}{E(n_1^2 - 1)} \tag{11}$$

The radial interference at the outside radius r_1 can be expressed as

$$\delta_1 = u_1 - u_{o1} \tag{12a}$$

where u_1 is the total radial deformation and u_{o1} is the deformation of the outside radius of the inner cylinder where

$$u_1 = \frac{1-\mu}{E} \frac{p_1}{(n_2^2 - 1)} r_1 + \frac{1+\mu}{E} \frac{r_2^2}{r_1} \frac{p_1}{(n_2^2 - 1)}$$

or

$$u_1 = \frac{1-\mu}{E} \frac{n_1^2 r_1 p_1}{(n_k^2 - n_1^2)} + \frac{1+\mu}{E} \frac{r_1 n_k^2 p_1}{(n_k^2 - n_1^2)}$$

$$u_{o1} = \frac{1-\mu}{E} \frac{r_1(p_o - n_1^2 p_1)}{n_1^2 - 1} + \frac{1+\mu}{E} \frac{r_1(p_o - p_1)}{n_1^2 - 1}$$

which will yield the solution

$$\delta_1 = \frac{2r_1}{E} \frac{n_1^2(p_o - p_1) - n_k^2(p_o - n_1^2 p_1)}{(n_k^2 - n_1^2)(n_1^2 - 1)} \tag{12b}$$

VII. Program for Multi-Region Pressure Vessel Using Maximum Shear Theory

1. Program Nomenclature

EN1(I)	=	n_1, ratio outside to inside radius of first ring (subscripted)	
ENK	=	n_k, total outside to inside radius ratio	
ENMAX	=	N_k, maximum total outside to inside radius ratio	
p_o	=	Internal pressure	$p_o \times 10^5$ psi
p_1	=	External pressure	$p_1 \times 10^5$ psi
RATIO(K)	=	p_o/τ (subscripted)	
TAU	=	Shear stress	$\tau \times 10^5$ psi
XK(J)	=	$K = (r - r_o)/(r_1 - r_o)$ (subscripted)	
XLDA	=	$\lambda = (r/r_o)^2$	

2. Program

The program written in Fortran II language is as follows.

```
    DIMENSION EN1(25), XK(5), RATIO(35)
    READ 2, (EN1(I), I=1, 25), (XK(J), J=1.4), (RATIO(K), K=1, 35)
  2 FORMAT (5F10.2)
    TAU=1.35
    DO 122 I=1, 25
    ENMAX=EN1(I)*EN1(I)
    PRINT 6, EN1(I), ENMAX, TAU
  6 FORMAT (1X, 4HN1=  , F5.1,10X,4HNK=  , F6.2,10X, 3HT=  , F5.2/)
    DO 122 J=1, 4
    TEMP=(XK(J)*(EN1(I)-1.0)+1.0
    XLDA=TEMP*TEMP
    PRINT 100
100 FORMAT (6X, 4HPO/T, 7X, 2HNK, 8X, 2HPO, 8X, 2HP1, 7X, 4HLDA //)
    PRINT 101. XLDA
101 FORMAT (45X, F6.3)
    DO 122 K=1, 35
    CODE=ENMAX-1.0
    BETA=XLDA*CODE+(1.0-RATIO(K))*ENMAX
    IF(BETA)122, 122, 8
  8 ENK=SQRTF(ENMAX*ENMAX/BETA)
    IF (ENK-ENMAX) 10,10,122
 10 IF (ENK-EN1(I)) 122, 12, 12
 12 PO=RATIO(K)*TAU
    P1=PO-TAU*XLDA*CODE/ENMAX
    PRINT 20, RATIO(K), ENK, PO, P1
 20 FORMAT (1X, 4F10.3)
122 CONTINUE
    CALL EXIT
    END
```

3. Input Data

n_1

1.6	1.7	1.8	1.9	2.0
2.1	2.2	.23	2.4	2.5
2.6	2.7	2.8	2.9	3.0
3.1	3.2	3.3	3.4	3.5
3.6	3.7	3.8	3.9	4.0

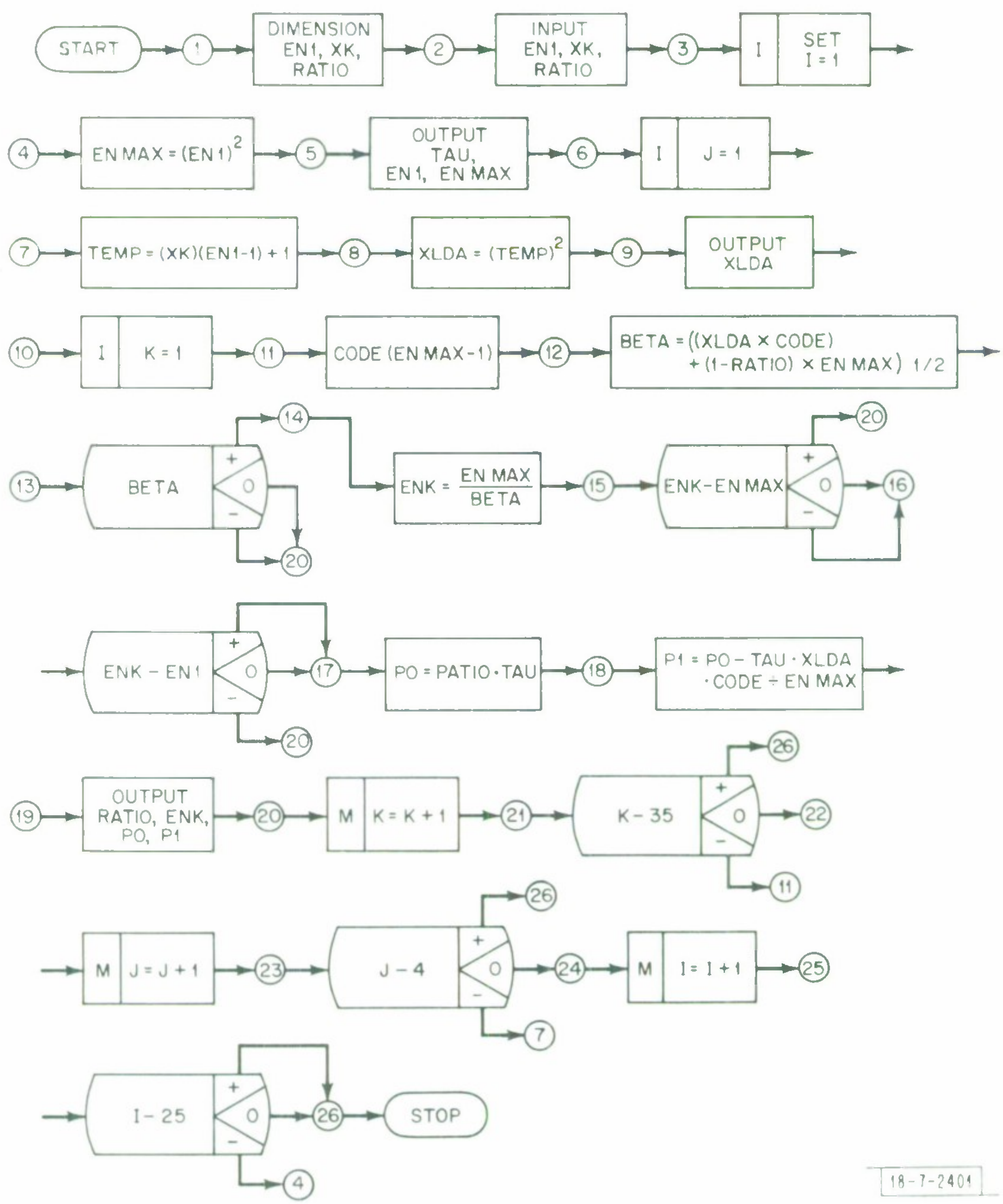
K

0.00	0.10	0.20	0.25

p_o/tau

1.1	1.2	1.3	1.4	1.5
1.6	1.7	1.8	1.9	2.0
2.1	2.2	2.3	2.4	2.5
2.6	2.7	2.8	2.9	3.0
3.1	3.2	3.3	3.4	3.5
3.6	3.7	3.8	3.9	4.0

4. Flow Diagram



Flow chart for two-region system.

VIII. Design Procedure

For the convenience of the designer, the tables in the Appendix are given to aid him in his choice of vessel dimensions. The material in this example has a .01 percent offset yield stress of 270,000 psi. If the problem is to design a pressure vessel with an inner diameter of .500 inch and to withstand pressures of 250,000 psi, the tables would give an enormous choice of pressure vessels that meet these requirements. If, however, a dimensional limit is given and a maximum O. D. of 2-1/2 inches or $N_k = 5$ is allowed, the following rules have to be applied:

a. Take λ with smallest K value. Thus, λ_{10} to be preferred over λ_{20}.

b. Take the largest p_o/τ value.

c. Choose dimension N_k as close as possible to n_1^2 value.

According to these rules, the problem is reduced to one of designing the pressure vessel where

$$p_o/\tau = 1.9$$

$$n_1 = 2.8$$

$$n_k = 4.990$$

$$p_o = 2.565 \times 10^5 \text{ psi}$$

$$p_1 = .925 \times 10^5 \text{ psi}$$

However, this solution does not exclude any of the other design possibilities that are given by the tables.

The preceding information yields the following vessel dimensions:

I.D. first ring = .5 inch

O.D. first ring = 1.4 inch = I.D. second ring

O.D. second ring = 2.495 inch or take 2.5 inch

The diametral interference determined from Eq. (12b) is .010 inch.

REFERENCES

1. T. E. Davidson, et al, "The Auto-Frettage Principle as Applied to High Strength Lightweight Gun Tubes," Technical Report WVTRI-5907, Watervliet Arsenal, Watervliet, N. Y.

2. S. Timoshenko, Strength of Materials (D. Van Nostrand, Inc., New York, 1956), Vol. II, 3rd Edition.

APPENDIX

$N_1 = 1.6$ $N_k = 2.56$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.7	1.678	.95	.122	1
.8	1.778	1.08	.257	
.9	1.900	1.215	.392	
1.0	2.050	1.35	.527	
1.1	2.242	1.485	.662	
1.2	2.501	1.62	.797	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
.7	1.612	.95	.020	1.124
.8	1.701	1.08	.155	
.9	1.806	1.215	.290	
1.0	1.933	1.35	.425	
1.1	2.092	1.485	.560	
1.2	2.298	1.62	.695	
1.3	2.579	1.755	.830	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
.8	1.629	1.08	.048	1.254
.9	1.721	1.215	.183	
1.0	1.830	1.35	.318	
1.1	1.963	1.485	.453	
1.2	2.130	1.62	.588	
1.3	2.348	1.755	.723	
1.4	2.651	1.89	.858	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
.9	1.681	1.215	.127	1.323
1.0	1.782	1.35	.262	
1.1	1.904	1.485	.397	
1.2	2.055	1.62	.532	
1.3	2.249	1.755	.667	
1.4	2.510	1.89	.802	

$N_1 = 1.7$ $N_k = 2.89$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.7	1.741	.95	.062	1
.8	1.840	1.08	.197	
.9	1.958	1.215	.332	
1.0	2.102	1.35	.467	
1.1	2.284	1.485	.602	
1.2	2.523	1.62	.737	
1.3	2.857	1.755	.872	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_o \times 10^5$ psi	λ_{10}
.8	1.745	1.08	.069	1.145
.9	1.845	1.215	.204	
1.0	1.965	1.35	.339	
1.1	2.111	1.485	.474	
1.2	2.295	1.62	.609	
1.3	2.538	1.755	.744	
1.4	2.878	1.89	.879	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_o \times 10^5$ psi	λ_{20}
.9	1.744	1.215	.067	1.30
1.0	1.844	1.35	.202	
1.1	1.963	1.485	.337	
1.2	2.108	1.62	.472	
1.3	2.292	1.755	.607	
1.4	2.534	1.89	.742	
1.5	2.873	2.025	.877	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_o \times 10^5$ psi	λ_{25}
1.0	1.789	1.35	.131	1.381
1.1	1.897	1.485	.266	
1.2	2.027	1.62	.401	
1.3	2.189	1.755	.536	
1.4	2.397	1.89	.671	
1.5	2.677	2.025	.806	
1.6	3.088	2.16	.941	

$N_1 = 1.8$ $N_k = 3.24$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.7	1.808	.95	.012	1
.8	1.907	1.08	.147	
.9	2.023	1.215	.282	
1.0	2.165	1.35	.417	
1.1	2.341	1.485	.552	
1.2	2.568	1.62	.687	
1.3	2.877	1.755	.822	
1.4	3.335	1.89	.957	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
.9	1.891	1.215	.127	1.166
1.0	2.005	1.35	.262	
1.1	2.142	1.485	.397	
1.2	2.312	1.62	.532	
1.3	2.530	1.755	.667	
1.4	2.825	1.89	.802	
1.5	3.253	2.025	.937	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.0	1.866	1.35	.094	1.346
1.1	1.975	1.485	.229	
1.2	2.106	1.62	.364	
1.3	2.267	1.755	.499	
1.4	2.471	1.89	.634	
1.5	2.743	2.025	.769	
1.6	3.131	2.16	.904	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.0	1.804	1.35	.006	1.44
1.1	1.902	1.485	.141	
1.2	2.018	1.62	.276	
1.3	2.158	1.755	.411	
1.4	2.332	1.89	.546	
1.5	2.557	2.025	.681	
1.6	2.862	2.16	.816	
1.7	3.311	2.295	.951	

$N_1 = 1.9$ $N_k = 3.61$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.8	1.978	1.08	.104	1
.9	2.094	1.215	.239	
1.0	2.235	1.35	.374	
1.1	2.407	1.485	.509	
1.2	2.627	1.62	.644	
1.3	2.921	1.755	.779	
1.4	3.343	1.89	.914	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
.9	1.940	1.215	.055	1.188
1.0	2.050	1.35	.190	
1.1	2.181	1.485	.325	
1.2	2.341	1.62	.460	
1.3	2.541	1.755	.595	
1.4	2.805	1.89	.730	
1.5	3.171	2.025	.865	
1.6	3.734	2.16	1.000	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.1	1.996	1.485	.126	1.392
1.2	2.116	1.62	.261	
1.3	2.261	1.755	.396	
1.4	2.440	1.89	.531	
1.5	2.670	2.025	.666	
1.6	2.980	2.16	.801	
1.7	3.432	2.295	.936	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.1	1.915	1.485	.021	1.50
1.2	2.020	1.62	.156	
1.3	2.145	1.755	.291	
1.4	2.297	1.89	.426	
1.5	2.485	2.025	.561	
1.6	2.730	2.16	.696	
1.7	3.064	2.295	.831	
1.8	3.562	2.43	.966	

$N_1 = 2$ $\qquad$ $N_k = 4$ $\qquad$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.8	2.0	1.08	.067	1
.9	2.169	1.215	.202	
1.0	2.3[illegible]9	1.35	.337	
1.1	2.4[illegible]1	1.485	.472	
1.2	2.697	1.62	.607	
1.3	2.981	1.755	.742	
1.4	3.381	1.89	.877	
1.5	4.000	2.025	1.012	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.0	2.099	1.35	.125	1.21
1.1	2.226	1.485	.260	
1.2	3.378	1.62	.395	
1.3	2.566	1.755	.530	
1.4	2.807	1.89	.665	
1.5	3.133	2.025	.800	
1.6	3.607	2.16	.935	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.1	2.020	1.485	.027	1.44
1.2	2.132	1.62	.162	
1.3	2.265	1.755	.297	
1.4	2.425	1.89	.432	
1.5	2.626	2.025	.567	
1.6	2.887	2.16	.702	
1.7	3.244	2.295	.837	
1.8	3.780	2.43	.972	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.2	2.028	1.62	.037	1.563
1.3	2.141	1.755	.172	
1.4	2.276	1.89	.307	
1.5	2.439	2.025	.442	
1.6	2.644	2.16	.577	
1.7	2.910	2.295	.712	
1.8	3.278	2.43	.847	
1.9	3.833	2.565	.982	

$N_1 = 2.1$ $\quad N_k = 4.41$ $\quad \tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.8	2.129	1.08	.036	1
.9	2.247	1.215	.171	
1.0	2.388	1.35	.306	
1.1	2.559	1.485	.441	
1.2	2.774	1.62	.576	
1.3	3.053	1.755	.711	
1.4	3.437	1.89	.846	
1.5	4.017	2.025	.981	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.0	2.152	1.35	.064	1.232
1.1	2.274	1.485	.199	
1.2	2.421	1.62	.334	
1.3	2.599	1.755	.469	
1.4	2.825	1.89	.604	
1.5	3.121	2.025	.739	
1.6	3.536	2.16	.874	
1.7	4.178	2.295	1.009	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.2	2.154	1.62	.067	1.488
1.3	2.277	1.755	.202	
1.4	2.424	1.89	.337	
1.5	2.604	2.025	.472	
1.6	2.830	2.16	.607	
1.7	3.128	2.295	.742	
1.8	3.547	2.43	.877	
1.9	4.195	2.565	1.012	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.3	2.146	1.755	.058	1.626
1.4	2.268	1.89	.193	
1.5	2.413	2.025	.328	
1.6	2.590	2.16	.463	
1.7	2.813	2.295	.598	
1.8	3.105	2.43	.733	
1.9	3.513	2.565	.868	
2.0	4.140	2.70	1.003	

$N_1 = 2.2$ $N_k = 4.84$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.8	2.207	1.08	.009	1
.9	2.328	1.215	.144	
1.0	2.470	1.35	.279	
1.1	2.642	1.485	.414	
1.2	2.856	1.62	.549	
1.3	3.132	1.755	.684	
1.4	3.508	1.89	.819	
1.5	4.062	2.025	.954	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.0	2.206	1.35	.007	1.254
1.1	2.326	1.485	.142	
1.2	2.468	1.62	.277	
1.3	2.639	1.755	.412	
1.4	2.852	1.89	.547	
1.5	3.127	2.025	.682	
1.6	3.501	2.16	.817	
1.7	4.051	2.295	.952	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.3	2.293	1.755	.108	1.538
1.4	2.429	1.89	.243	
1.5	2.592	2.025	.378	
1.6	2.793	2.16	.513	
1.7	3.050	2.295	.648	
1.8	3.394	2.43	.783	
1.9	3.888	2.565	.918	
2.0	4.688	2.70	1.053	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.4	2.268	1.89	.080	1.69
1.5	2.399	2.025	.215	
1.6	2.556	2.16	.350	
1.7	2.748	2.295	.485	
1.8	2.992	2.43	.620	
1.9	3.314	2.565	.755	
2.0	3.768	2.7	.890	
2.1	4.483	2.835	1.025	

$N_1 = 2.3$ $\quad N_k = 5.29$ $\quad \tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	2.410	1.215	.120	1
1.0	2.554	1.35	.255	
1.1	2.728	1.485	.390	
1.2	2.943	1.62	.525	
1.3	3.218	1.755	.660	
1.4	3.588	1.89	.795	
1.5	4.125	2.025	.930	
1.6	5.008	2.16	1.065	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.1	2.378	1.485	.087	1.277
1.2	2.516	1.62	.222	
1.3	2.682	1.755	.357	
1.4	2.885	1.89	.492	
1.5	3.143	2.025	.627	
1.6	3.485	2.16	.762	
1.7	3.970	2.295	.897	
1.8	4.738	2.43	1.032	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.3	2.324	1.755	.027	1.578
1.4	2.452	1.89	.162	
1.5	2.605	2.025	.297	
1.6	2.790	2.16	.432	
1.7	3.021	2.295	.567	
1.8	3.321	2.43	.702	
1.9	3.733	2.565	.837	
2.0	4.349	2.7	.972	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.6	2.418	2.16	.128	1.856
1.7	2.563	2.295	.263	
1.8	2.739	2.43	.398	
1.9	2.957	2.565	.533	
2.0	3.236	2.7	.668	
2.1	3.613	2.835	.803	
2.2	4.164	2.97	.938	
2.3	5.078	3.105	1.073	

$N_1 = 2.4$ $N_k = 5.76$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	2.494	1.215	.099	1
1.0	2.640	1.35	.234	
1.1	2.816	1.485	.369	
1.2	3.032	1.62	.504	
1.3	3.308	1.755	.639	
1.4	3.675	1.89	.774	
1.5	4.201	2.025	.909	
1.6	5.044	2.16	1.044	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.1	2.431	1.485	.035	1.3
1.2	2.567	1.62	.170	
1.3	2.727	1.755	.305	
1.4	2.923	1.89	.440	
1.5	3.167	2.025	.575	
1.6	3.485	2.16	.710	
1.7	3.923	2.295	.845	
1.8	4.582	2.43	.980	
1.9	5.749	2.565	1.115	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.4	2.458	1.89	.063	1.638
1.5	2.598	2.025	.198	
1.6	2.765	2.16	.333	
1.7	2.969	2.295	.468	
1.8	3.226	2.43	.603	
1.9	3.563	2.565	.738	
2.0	4.036	2.7	.873	
2.1	4.766	2.835	1.008	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.6	2.521	2.16	.126	1.823
1.7	2.672	2.295	.261	
1.8	2.855	2.43	.396	
1.9	3.082	2.565	.531	
2.0	3.372	2.7	.666	
2.1	3.764	2.835	.801	
2.2	4.335	2.97	.936	
2.3	5.281	3.105	1.071	

N_1 = 2.5 $\qquad$ N_k = 6.250 $\qquad$ τ = 1.35 x 10^5 psi

p_o/τ	n_k	p_o x 10^5 psi	p_1 x 10^5 psi	λ_0
.9	2.579	1.215	.081	1
1.0	2.728	1.35	.216	
1.1	2.906	1.485	.351	
1.2	3.125	1.62	.486	
1.3	3.402	1.755	.621	
1.4	3.769	1.89	.756	
1.5	4.287	2.025	.891	
1.6	5.103	2.16	1.026	

p_o/τ	n_k	p_o x 10^5 psi	p_1 x 10^5 psi	λ_{10}
1.2	2.619	1.62	.120	1.392
1.3	2.776	1.755	.255	
1.4	2.964	1.89	.390	
1.5	3.197	2.025	.525	
1.6	3.496	2.16	.660	
1.7	3.898	2.295	.795	
1.8	4.481	2.43	.930	
1.9	5.438	2.565	1.065	

p_o/τ	n_k	p_o x 10^5 psi	p_1 x 10^5 psi	λ_{20}
1.5	2.607	2.025	.109	1.69
1.6	2.761	2.16	.244	
1.7	2.947	2.295	.379	
1.8	3.176	2.43	.514	
1.9	3.468	2.565	.649	
2.0	3.859	2.7	.784	
2.1	4.422	2.835	.919	
2.2	5.335	2.97	1.054	

p_o/τ	n_k	p_o x 10^5 psi	p_1 x 10^5 psi	λ_{25}
1.6	2.516	2.16	.017	1.89
1.7	2.654	2.295	.152	
1.8	2.817	2.43	.287	
1.9	3.015	2.565	.422	
2.0	3.261	2.7	.557	
2.1	3.580	2.835	.692	
2.2	4.016	2.97	.827	
2.3	4.662	3.105	.962	
2.4	5.772	3.24	1.097	

$N_1 = 2.6$ $N_k = 6.76$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	2.665	1.215	.065	1
1.0	2.817	1.35	.200	
1.1	2.998	1.485	.335	
1.2	3.220	1.62	.470	
1.3	3.499	1.755	.605	
1.4	3.867	1.89	.740	
1.5	4.382	2.025	.875	
1.6	5.179	2.16	1.010	
1.7	6.667	2.295	1.145	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.2	2.672	1.62	.072	1.346
1.3	2.825	1.755	.207	
1.4	3.008	1.89	.342	
1.5	3.233	2.025	.477	
1.6	3.516	2.16	.612	
1.7	3.889	2.295	.747	
1.8	4.414	2.43	.882	
1.9	5.233	2.565	1.017	
2.0	6.784	2.70	1.152	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.6	2.736	2.16	.131	1.764
1.7	2.901	2.295	.266	
1.8	3.101	2.43	.401	
1.9	3.348	2.565	.536	
2.0	3.666	2.70	.671	
2.1	4.095	2.835	.806	
2.2	4.723	2.97	.941	
2.3	5.770	3.105	1.076	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.7	2.640	2.295	.040	1.96
1.8	2.787	2.43	.175	
1.9	2.963	2.565	.310	
2.0	3.176	2.70	.445	
2.1	3.444	2.835	.580	
2.2	3.792	2.97	.715	
2.3	4.274	3.105	.850	
2.4	5.003	3.24	.985	
2.5	6.305	3.375	1.120	

$N_1 = 2.7$ $N_k = 7.29$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	2.752	1.215	.050	1
1.0	2.907	1.35	.185	
1.1	3.091	1.485	.320	
1.2	3.316	1.62	.455	
1.3	3.599	1.755	.590	
1.4	3.969	1.89	.725	
1.5	4.482	2.025	.860	
1.6	4.267	2.16	.995	
1.7	6.691	2.295	1.130	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.2	2.726	1.62	.025	1.369
1.3	2.876	1.755	.160	
1.4	3.055	1.89	.295	
1.5	3.271	2.025	.430	
1.6	3.542	2.16	.565	
1.7	3.892	2.295	.700	
1.8	4.373	2.43	.835	
1.9	5.092	2.565	.970	
2.0	6.343	2.7	1.105	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.6	2.771	2.16	.068	1.796
1.7	2.929	2.295	.203	
1.8	3.118	2.43	.338	
1.9	3.350	2.565	.473	
2.0	3.642	2.7	.608	
2.1	4.027	2.835	.743	
2.2	4.566	2.97	.878	
2.3	5.404	3.105	1.013	
2.4	6.980	3.24	1.148	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.8	2.767	2.43	.064	2.031
1.9	2.924	2.565	.199	
2.0	3.113	2.7	.334	
2.1	3.343	2.835	.469	
2.2	3.633	2.97	.604	
2.3	4.014	3.105	.739	
2.4	4.548	3.24	.874	
2.5	5.374	3.375	1.009	
2.6	6.916	3.51	1.144	

$N_1 = 2.8$ $N_k = 7.84$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	2.839	1.215	.037	1
1.0	2.998	1.35	.172	
1.1	3.186	1.485	.307	
1.2	3.415	1.62	.442	
1.3	3.701	1.755	.577	
1.4	4.074	1.89	.712	
1.5	4.588	2.025	.847	
1.6	5.364	2.16	.982	
1.7	6.743	2.295	1.117	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.3	2.927	1.755	.115	1.3924
1.4	3.102	1.89	.250	
1.5	3.312	2.025	.385	
1.6	3.571	2.16	.520	
1.7	3.902	2.295	.655	
1.8	4.348	2.43	.790	
1.9	4.990	2.565	.925	
2.0	6.041	2.7	1.060	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.7	2.929	2.295	.117	1.8496
1.8	3.104	2.43	.252	
1.9	3.314	2.565	.387	
2.0	3.574	2.7	.522	
2.1	3.907	2.835	.657	
2.2	4.353	2.97	.792	
2.3	4.999	3.105	.927	
2.4	6.057	3.24	1.062	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.9	2.897	2.565	.089	2.1025
2.0	3.065	2.7	.224	
2.1	3.267	2.835	.359	
2.2	3.516	2.97	.494	
2.3	3.831	3.105	.629	
2.4	4.249	3.24	.764	
2.5	4.843	3.375	.899	
2.6	5.784	3.51	1.034	
2.7	7.640	3.645	1.169	

$N_1 = 2.9$ $N_k = 8.41$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	2.928	1.215	.026	1
1.0	3.089	1.35	.161	
1.1	3.281	1.485	.296	
1.2	3.514	1.62	.431	
1.3	3.804	1.755	.566	
1.4	4.181	1.89	.701	
1.5	4.698	2.025	.836	
1.6	5.470	2.16	.971	
1.7	6.815	2.295	1.106	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.3	2.979	1.755	.071	1.4161
1.4	3.150	1.89	.206	
1.5	3.354	2.025	.341	
1.6	3.603	2.16	.476	
1.7	3.918	2.295	.611	
1.8	4.334	2.43	.746	
1.9	4.918	2.565	.881	
2.0	5.827	2.7	1.016	
2.1	7.545	2.835	1.151	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.7	2.933	2.295	.030	1.9044
1.8	3.095	2.43	.165	
1.9	3.288	2.565	.300	
2.0	3.522	2.7	.435	
2.1	3.815	2.835	.570	
2.2	4.195	2.97	.705	
2.3	4.717	3.105	.840	
2.4	5.501	3.24	.975	
2.5	6.875	3.375	1.110	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
1.9	2.940	2.565	.036	2.126
2.0	3.103	2.7	.171	
2.1	3.298	2.835	.306	
2.2	3.534	2.97	.441	
2.3	3.830	3.105	.576	
2.4	4.216	3.24	.711	
2.5	4.747	3.375	.846	
2.6	5.548	3.51	.981	
2.7	6.968	3.645	1.116	

$N_1 = 3$ $N_k = 9$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	3.017	1.215	.015	1
1.0	3.182	1.35	.150	
1.1	3.378	1.485	.285	
1.2	3.614	1.62	.420	
1.3	3.909	1.755	.555	
1.4	4.291	1.89	.690	
1.5	4.811	2.025	.825	
1.6	5.582	2.16	.960	
1.7	6.903	2.295	1.095	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.3	3.030	1.755	.027	1.44
1.4	3.198	1.89	.162	
1.5	3.397	2.025	.297	
1.6	3.638	2.16	.432	
1.7	3.939	2.295	.567	
1.8	4.330	2.43	.702	
1.9	4.867	2.565	.837	
2.0	5.669	2.7	.972	
2.1	7.071	2.835	1.107	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.8	3.091	2.43	.078	1.96
1.9	3.269	2.565	.213	
2.0	3.482	2.7	.348	
2.1	3.744	2.835	.483	
2.2	4.074	2.97	.618	
2.3	4.511	3.105	.753	
2.4	5.128	3.24	.888	
2.5	6.096	3.375	1.023	
2.6	7.955	3.51	1.158	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.0	3.000	2.7	.000	2.25
2.1	3.162	2.835	.135	
2.2	3.354	2.97	.270	
2.3	3.586	3.105	.405	
2.4	3.873	3.24	.540	
2.5	4.243	3.375	.675	
2.6	4.743	3.51	.810	
2.7	5.477	3.645	.945	
2.8	6.708	3.78	1.080	

$N_1 = 3.1$ $N_k = 9.61$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	3.106	1.215	.005	1
1.0	3.275	1.35	.140	
1.1	3.475	1.485	.275	
1.2	3.716	1.62	.410	
1.3	4.016	1.755	.545	
1.4	4.402	1.89	.680	
1.5	4.927	2.025	.815	
1.6	5.698	2.16	.950	
1.7	7.003	2.295	1.085	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.3	3.096	1.755	- .004	1.4541
1.4	3.263	1.89	.131	
1.5	3.460	2.025	.266	
1.6	3.698	2.16	.401	
1.7	3.993	2.295	.536	
1.8	4.372	2.43	.671	
1.9	4.885	2.565	.806	
2.0	5.634	2.7	.941	
2.1	6.884	2.835	1.076	
2.2	9.669	2.97	1.211	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.8	3.090	2.43	- .009	2.0164
1.9	3.256	2.565	.126	
2.0	3.452	2.7	.261	
2.1	3.688	2.835	.396	
2.2	3.980	2.97	.531	
2.3	4.356	3.105	.666	
2.4	4.862	3.24	.801	
2.5	5.599	3.375	.936	
2.6	6.821	3.51	1.071	
2.7	9.496	3.645	1.206	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.1	3.126	2.835	.022	2.325625
2.2	3.298	2.97	.157	
2.3	3.502	3.105	.292	
2.4	3.749	3.24	.427	
2.5	4.058	3.375	.562	
2.6	4.458	3.51	.697	
2.7	5.005	3.645	.832	
2.8	5.821	3.78	.967	
2.9	7.234	3.915	1.102	

$N_1 = 3.2$ $N_k = 10.24$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	3.196	1.215	- .003	1
1.0	3.369	1.35	.132	
1.1	3.572	1.485	.267	
1.2	3.818	1.62	.402	
1.3	4.123	1.755	.537	
1.4	4.515	1.89	.672	
1.5	5.045	2.025	.807	
1.6	5.820	2.16	.942	
1.7	7.114	2.295	1.077	
1.8	10.003	2.43	1.212	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.4	3.295	1.89	.077	1.4884
1.5	3.485	2.025	.212	
1.6	3.712	2.16	.347	
1.7	3.991	2.295	.482	
1.8	4.342	2.43	.617	
1.9	4.808	2.565	.752	
2.0	5.464	2.7	.887	
2.1	6.491	2.835	1.022	
2.2	8.461	2.97	1.157	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
1.9	3.247	2.656	.039	2.0736
2.0	3.429	2.7	.174	
2.1	3.644	2.835	.309	
2.2	3.906	2.97	.444	
2.3	4.234	3.105	.579	
2.4	4.662	3.24	.714	
2.5	5.253	3.375	.849	
2.6	6.146	3.51	.984	
2.7	7.736	3.645	1.119	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.2	3.253	2.97	.043	2.4025
2.3	3.435	3.105	.178	
2.4	3.652	3.24	.313	
2.5	3.916	3.375	.448	
2.6	4.246	3.51	.583	
2.7	4.678	3.645	.718	
2.8	5.276	3.78	.853	
2.9	6.183	3.915	.988	
3.0	7.810	4.05	1.123	

$N_1 = 3.3$ $N_k = 10.89$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
.9	3.287	1.215	- .011	1
1.0	3.463	1.35	.124	
1.1	3.671	1.485	.259	
1.2	3.921	1.62	.394	
1.3	4.232	1.755	.529	
1.4	4.629	1.89	.664	
1.5	5.165	2.025	.799	
1.6	5.945	2.16	.934	
1.7	7.233	2.295	1.069	
1.8	10.034	2.43	1.204	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.4	3.344	1.89	.035	1.5129
1.5	3.530	2.025	.170	
1.6	3.751	2.16	.305	
1.7	4.020	2.295	.440	
1.8	4.356	2.43	.575	
1.9	4.793	2.565	.710	
2.0	5.396	2.7	.845	
2.1	6.305	2.835	.980	
2.2	7.912	2.97	1.115	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
2.0	3.411	2.7	.087	2.1316
2.1	3.609	2.835	.222	
2.2	3.847	2.97	.357	
2.3	4.138	3.105	.492	
2.4	4.508	3.24	.627	
2.5	4.999	3.375	.762	
2.6	5.694	3.51	.897	
2.7	6.795	3.645	1.032	
2.8	8.953	3.78	1.167	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.3	3.381	3.105	.064	2.480625
2.4	3.573	3.24	.199	
2.5	3.803	3.375	.334	
2.6	4.084	3.51	.469	
2.7	4.438	3.645	.604	
2.8	4.904	3.78	.739	
2.9	5.556	3.915	.874	
3.0	6.563	4.050	1.009	
3.1	8.441	4.185	1.144	

$N_1 = 3.4$ $N_k = 11.56$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
1.0	3.557	1.35	.117	1
1.1	3.770	1.485	.252	
1.2	4.025	1.62	.387	
1.3	4.341	1.755	.522	
1.4	4.745	1.89	.657	
1.5	5.287	2.025	.792	
1.6	6.072	2.16	.927	
1.7	7.358	2.295	1.062	
1.8	10.092	2.43	1.197	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.4	3.392	1.89	- .006	1.5376
1.5	3.575	2.025	.129	
1.6	3.790	2.16	.264	
1.7	4.051	2.295	.399	
1.8	4.373	2.43	.534	
1.9	4.786	2.565	.669	
2.0	5.345	2.7	.804	
2.1	6.161	2.835	.939	
2.2	7.517	2.97	1.074	
2.3	10.513	3.105	1.209	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
2.0	3.398	2.7	- .001	2.1904
2.1	3.582	2.835	.134	
2.2	3.799	2.97	.269	
2.3	4.061	3.105	.404	
2.4	4.386	3.24	.539	
2.5	4.804	3.375	.674	
2.6	5.370	3.51	.809	
2.7	6.198	3.645	.944	
2.8	7.585	3.78	1.079	
2.9	10.703	3.915	1.214	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.4	3.510	3.24	.083	2.56
2.5	3.713	3.375	.218	
2.6	3.956	3.51	.353	
2.7	4.255	3.645	.488	
2.8	4.633	3.78	.623	
2.9	5.134	3.915	.758	
3.0	5.843	4.05	.893	
3.1	6.961	4.185	1.028	
3.2	9.134	4.32	1.163	

$N_1 = 3.5$ $N_k = 12.25$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
1.0	3.652	1.35	.110	1
1.1	3.869	1.485	.245	
1.2	4.129	1.62	.380	
1.3	4.451	1.755	.515	
1.4	4.861	1.89	.650	
1.5	5.411	2.025	.785	
1.6	6.203	2.16	.920	
1.7	7.490	2.295	1.055	
1.8	10.173	2.43	1.190	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.5	3.620	2.025	.088	1.5625
1.6	3.830	2.16	.223	
1.7	4.083	2.295	.358	
1.8	4.392	2.43	.493	
1.9	4.785	2.565	.628	
2.0	5.307	2.7	.763	
2.1	6.048	2.835	.898	
2.2	7.221	2.97	1.033	
2.3	9.528	3.105	1.168	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
2.1	3.560	2.835	.045	2.25
2.2	3.760	2.97	.180	
2.3	3.998	3.105	.315	
2.4	4.288	3.24	.450	
2.5	4.651	3.375	.585	
2.6	5.125	3.51	.720	
2.7	5.783	3.645	.855	
2.8	6.782	3.78	.990	
2.9	8.582	3.915	1.125	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.5	3.639	3.375	.101	2.640625
2.6	3.853	3.51	.236	
2.7	4.110	3.645	.371	
2.8	4.427	3.78	.506	
2.9	4.830	3.915	.641	
3.0	5.368	4.05	.776	
3.1	6.139	4.185	.911	
3.2	7.378	4.32	1.047	
3.3	9.897	4.455	1.181	

$N_1 = 3.6$ $N_k = 12.96$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
1.0	3.747	1.35	.104	1
1.1	3.969	1.485	.239	
1.2	4.234	1.62	.374	
1.3	4.562	1.755	.509	
1.4	4.979	1.89	.644	
1.5	5.536	2.025	.779	
1.6	6.336	2.16	.914	
1.7	7.626	2.295	1.049	
1.8	10.271	2.43	1.184	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.5	3.665	2.025	.047	1.5876
1.6	3.871	2.16	.182	
1.7	4.116	2.295	.317	
1.8	4.414	2.43	.452	
1.9	4.789	2.565	.587	
2.0	5.279	2.7	.722	
2.1	5.958	2.835	.857	
2.2	6.992	2.97	.992	
2.3	8.860	3.105	1.127	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
2.2	3.729	2.97	.092	2.3104
2.3	3.946	3.105	.227	
2.4	4.207	3.24	.362	
2.5	4.528	3.375	.497	
2.6	4.935	3.51	.632	
2.7	5.476	3.645	.767	
2.8	6.247	3.78	.902	
2.9	7.472	3.915	1.037	
3.0	9.904	4.05	1.172	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.6	3.769	3.51	.118	2.7225
2.7	3.994	3.645	.253	
2.8	4.265	3.78	.388	
2.9	4.600	3.915	.523	
3.0	5.029	4.05	.658	
3.1	5.606	4.185	.793	
3.2	6.441	4.320	.928	
3.3	7.811	4.455	1.063	
3.4	10.736	4.59	1.198	

$N_1 = 3.7$ $N_k = 13.69$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
1.0	3.843	1.35	.099	1
1.1	4.069	1.485	.234	
1.2	4.340	1.62	.369	
1.3	4.673	1.755	.504	
1.4	5.097	1.89	.639	
1.5	5.663	2.025	.774	
1.6	6.471	2.16	.909	
1.7	7.767	2.295	1.044	
1.8	10.384	2.43	1.179	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.5	3.709	2.025	.007	1.6129
1.6	3.911	2.16	.142	
1.7	4.149	2.295	.277	
1.8	4.438	2.43	.412	
1.9	4.796	2.565	.547	
2.0	5.259	2.7	.682	
2.1	5.886	2.835	.817	
2.2	6.811	2.97	.952	
2.3	8.377	3.105	1.087	
2.4	11.999	3.24	1.222	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
2.2	3.706	2.97	.005	2.3696
2.3	3.908	3.105	.140	
2.4	4.146	3.24	.275	
2.5	4.433	3.375	.410	
2.6	4.791	3.51	.545	
2.7	5.251	3.645	.680	
2.8	5.876	3.78	.815	
2.9	6.795	3.915	.950	
3.0	8.347	4.05	1.085	
3.1	11.910	4.185	1.220	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.6	3.699	3.51	- .001	2.805625
2.7	3.899	3.645	.134	
2.8	4.135	3.78	.269	
2.9	4.420	3.915	.404	
3.0	4.774	4.05	.539	
3.1	5.229	4.185	.674	
3.2	5.845	4.32	.809	
3.3	6.748	4.455	.944	
3.4	8.259	4.59	1.079	
3.5	11.661	4.725	1.214	

$N_1 = 3.8$ $N_k = 14.44$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
1.0	3.939	1.35	.093	1
1.1	4.169	1.485	.228	
1.2	4.445	1.62	.363	
1.3	4.785	1.755	.498	
1.4	5.216	1.89	.633	
1.5	5.790	2.025	.768	
1.6	6.607	2.16	.903	
1.7	7.911	2.295	1.038	
1.8	10.509	2.43	1.173	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.6	3.951	2.16	.101	1.6384
1.7	4.184	2.295	.236	
1.8	4.463	2.43	.371	
1.9	4.807	2.565	.506	
2.0	5.245	2.7	.641	
2.1	5.829	2.835	.776	
2.2	6.666	2.97	.911	
2.3	8.012	3.105	1.046	
2.4	10.751	3.24	1.181	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
2.3	3.868	3.105	.047	2.4336
2.4	4.086	3.24	.182	
2.5	4.344	3.375	.317	
2.6	4.660	3.51	.452	
2.7	5.055	3.645	.587	
2.8	5.572	3.78	.722	
2.9	6.289	3.915	.857	
3.0	7.381	4.05	.992	
3.1	9.353	4.185	1.127	
3.2	14.897	4.320	1.262	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.7	3.819	3.645	.014	2.89
2.8	4.028	3.78	.149	
2.9	4.276	3.915	.284	
3.0	4.575	4.05	.419	
3.1	4.948	4.185	.554	
3.2	5.429	4.320	.689	
3.3	6.086	4.455	.824	
3.4	7.058	4.59	.959	
3.5	8.721	4.725	1.094	
3.6	12.676	4.86	1.229	

$N_1 = 3.9$ $N_k = 15.21$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
1.0	4.035	1.35	.089	1
1.1	4.270	1.485	.224	
1.2	4.551	1.62	.359	
1.3	4.897	1.755	.494	
1.4	5.336	1.89	.629	
1.5	5.918	2.025	.764	
1.6	6.746	2.16	.899	
1.7	8.058	2.295	1.034	
1.8	10.644	2.43	1.169	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.6	3.991	2.16	.061	1.6641
1.7	4.219	2.295	.196	
1.8	4.489	2.43	.331	
1.9	4.820	2.565	.466	
2.0	5.236	2.7	.601	
2.1	5.784	2.835	.736	
2.2	6.548	2.97	.871	
2.3	7.728	3.105	1.006	
2.4	9.916	3.24	1.141	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
2.4	4.039	3.24	.091	2.4964
2.5	4.275	3.375	.226	
2.6	4.558	3.51	.361	
2.7	4.905	3.645	.496	
2.8	5.346	3.78	.631	
2.9	5.932	3.915	.766	
3.0	6.766	4.05	.901	
3.1	8.092	4.185	1.036	
3.2	10.723	4.320	1.171	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.8	3.938	3.78	.026	2.97625
2.9	4.156	3.915	.161	
3.0	4.414	4.05	.296	
3.1	4.727	4.185	.431	
3.2	5.118	4.32	.566	
3.3	5.626	4.455	.701	
3.4	6.322	4.590	.836	
3.5	7.363	4.725	.971	
3.6	9.178	4.860	1.106	
3.7	13.739	4.995	1.241	

$N_1 = 4$ $N_k = 16$ $\tau = 1.35 \times 10^5$ psi

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_0
1.0	4.131	1.35	.084	1
1.1	4.371	1.485	.219	
1.2	4.658	1.62	.354	
1.3	5.010	1.755	.489	
1.4	5.456	1.89	.624	
1.5	6.047	2.025	.759	
1.6	6.885	2.16	.894	
1.7	8.208	2.295	1.029	
1.8	10.787	2.43	1.164	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{10}
1.6	4.032	2.16	.021	1.69
1.7	4.253	2.295	.156	
1.8	4.516	2.43	.291	
1.9	4.835	2.565	.426	
2.0	5.233	2.7	.561	
2.1	5.747	2.835	.696	
2.2	6.452	2.97	.831	
2.3	7.501	3.105	.966	
2.4	9.316	3.24	1.101	
2.5	13.771	3.375	1.236	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{20}
2.4	4.000	3.24	.000	2.56
2.5	4.216	3.375	.135	
2.6	4.472	3.51	.270	
2.7	4.781	3.645	.405	
2.8	5.164	3.78	.540	
2.9	5.657	3.915	.675	
3.0	6.325	4.05	.810	
3.1	7.303	4.185	.945	
3.2	8.944	4.32	1.080	
3.3	12.649	4.455	1.215	

p_o/τ	n_k	$p_o \times 10^5$ psi	$p_1 \times 10^5$ psi	λ_{25}
2.9	4.059	3.915	.039	3.0625
3.0	4.286	4.05	.174	
3.1	4.555	4.185	.309	
3.2	4.883	4.32	.444	
3.3	5.293	4.455	.579	
3.4	5.828	4.59	.714	
3.5	6.566	4.725	.849	
3.6	7.682	4.86	.984	
3.7	9.670	4.995	1.119	
3.8	15.002	5.13	1.254	

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13. ABSTRACT

A method is outlined for multi-region pressure vessels design calculations using the maximum shear theory. This treatment is employed due to the simplicity of the method and because the results are quite conservative for both ductile and brittle materials.

A procedure for obtaining an optimum design has been given for a desired percentage of auto-frettage on the inner wall of the pressure vessel. A computer program has been written in Fortran II language and the various design possibilities have been executed by IBM-1620 computer.

14. KEY WORDS

pressure vessels   shear theory